# **MULTIMEDIA UNIVERSITY**

## FINAL EXAMINATION

TRIMESTER 1, 2019/2020

### **DMT5131 – MATHEMATICAL TECHNIQUES 1**

(For DIT students only)

14 OCTOBER 2019 02.30 pm - 04.30 pm (2 Hours)

#### INSTRUCTIONS TO STUDENT

- 1. This question paper consists of 4 pages (2 pages with 3 questions and 2 pages of Appendix). Key formulae are given in the Appendix.
- 2. Answer ALL questions.
- 3. Write your answers in the answer booklet provided.
- 4. All necessary working steps must be shown.

#### **QUESTION 1**

a) Solve the equation of  $\sqrt{7x+9} = x+3$ . (2 marks)

b) Solve the inequality  $\frac{x-4}{(x+2)(2x+1)} < 0$  and express your answer in interval notation. (3 marks)

c) Given a quadratic function of  $f(x) = 2(x-2)^2 - 18$ .

i) State the vertex of the function. (0.5 mark)

ii) Find the coordinates of x-intercepts. (3 marks)

iii) Find the coordinate of y-intercept. (1.5 marks)

[TOTAL 10 MARKS]

#### **QUESTION 2**

a) Find matrix Q if  $4Q + \begin{bmatrix} 2 & -1 & 0 \\ 4 & 7 & -3 \end{bmatrix}^T = \begin{bmatrix} 10 & 12 \\ -3 & -1 \\ 4 & 9 \end{bmatrix}$  (2.5 marks)

b) Find value of x for which the following matrix is not invertible. (2 marks)  $\begin{bmatrix} x-2 & 2 \\ x & 3 \end{bmatrix}$ 

- c) Jenny enjoys making desserts for her neighbours. Last month, she made 3 batches of cream puff cake and 5 batches of cookies, which used a total of 22 eggs. The month before, she baked 2 batches of cream puff cake and 2 batches of cookies, which required a total of 12 eggs.
  - i) Represent the above information in the form of AX = B. (1.5 marks)

ii) Find  $A^{-1}$ . (2.5 marks)

d) The matrix given below is in the form of AX = B.

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & -5 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ -28 \\ 24 \end{bmatrix}$$

- i) From the information given, write down a system of linear equation. (1.5 marks)
- ii) Use Cramer's Rule to solve for y.

(5 marks)

[TOTAL 15 MARKS]

Continued...

#### **QUESTION 3**

- a) Given the first term of the sequence is -3. List the first four terms for the general sequence form of  $a_n = \frac{a_{n-1} + 4}{2^n}$ . (2.5 marks)
- b) Sally played an adventure game named Catching Dragon. She scored 300 points for capturing her first dragon and then 700 points for capturing her  $n^{th}$  dragon. While the total number of points for capturing all n dragon was 8500.
  - i) Given that the number of points that Sally scored for capturing each successive dragon formed an arithmetic sequence, find the value of n. (2 marks)
  - ii) Find the difference, d for each dragon captured. (2 marks)
- c) Given that the 3<sup>rd</sup> term of geometric sequence is  $-\frac{9}{4}$  and common ratio is  $\frac{3}{4}$ .
  - i) Find the 1<sup>st</sup> term. (1.5 marks)
  - ii) Find the sum of 13 terms. Leave your answer in 4 decimal places.
    (1.5 marks)
  - iii) Evaluate the sum to infinity. (1 mark)
- d) Expand  $(2x-3y)^3$  using the Binomial Theorem. (4.5 marks)

[TOTAL 15 MARKS]

#### APPENDIX - KEY FORMULA

Completing the square:  $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$ 

Quadratic formula: If  $ax^2 + bx + c = 0$  where  $a \ne 0$ , then,  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 

Standard form of a quadratic function:  $f(x) = a(x - h)^2 + k$ ,  $a \neq 0$ 

Determinant of a 2 × 2 matrix	Determinant of a 3 × 3 matrix
$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$	$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

Inverse of a 2 × 2 matrix

If 
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
,

then  $A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ ,

where  $ad - bc \neq 0$ .

Cramer's Rule for 2 × 2 matrix	Cramer's Rule for 3 × 3 matrix
then $x = \begin{vmatrix} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{vmatrix}$ then $x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \\ a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ and $y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \\ a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ , where $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$	$a_{1}x + b_{1}y + c_{1}z = d_{1}$ If $a_{2}x + b_{2}y + c_{2}z = d_{2}$ $a_{3}x + b_{3}y + c_{3}z = d_{3}$ then $x = \frac{D_{x}}{D}$ , $y = \frac{D_{y}}{D}$ , $z = \frac{D_{z}}{D}$ where $D = \begin{vmatrix} a_{1} & b_{1} & c_{1} \\ a_{2} & b_{2} & c_{2} \\ a_{3} & b_{3} & c_{3} \end{vmatrix}$ , $D_{x} = \begin{vmatrix} d_{1} & b_{1} & c_{1} \\ d_{2} & b_{2} & c_{2} \\ d_{3} & b_{3} & c_{3} \end{vmatrix}$ $D_{y} = \begin{vmatrix} a_{1} & d_{1} & c_{1} \\ a_{2} & d_{2} & c_{2} \\ a_{3} & d_{3} & c_{3} \end{vmatrix}$ , $D_{z} = \begin{vmatrix} a_{1} & b_{1} & d_{1} \\ a_{2} & b_{2} & d_{2} \\ a_{3} & b_{3} & d_{3} \end{vmatrix}$

Arithmetic sequence	Geometric sequence
$a_n = a_1 + (n-1)d$	$a_n = a_1 r^{n-1}, S_n = \frac{a_1 (1 - r^n)}{1 - r}$
$S_n = \frac{n}{2} (a_1 + a_n)$	$S_{\infty} = \frac{a_1}{1-r},  r  < 1$

#### Binomial Theorem

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k; \quad n \ge 1$$

The  $r^{\text{th}}$  term of the expansion of  $(a+b)^n$  is  $\binom{n}{r-1}a^{n-r+1}b^{r-1}$ .